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AGGLOMERATION OF ALUMINA PARTICLES IN THE FLOW OF A METALLIZED PROPELLANT ROCKET NOZZLE

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Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*<u>ye</u> initially, after vowels, and after ъ, ь; <u>e</u> elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

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RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}

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rot	curl
lg	log

AGGLOMERATION OF ALUMINA PARTICLES IN THE FLOW OF A METALLIZED PROPELLANT ROCKET NOZZLE*

Paul Kuentzmann

ABSTRACT. The agglomeration of liquid alumina particles takes place in the convergent region and at the throat of the nozzle; it leads to an increase of performance losses related to the velocity and temperature differences between the condensed and gaseous phases.

The problem is first schematized; then the fundamental equations of the problem are established that give the evolution of the distribution law of the particle population, and of the variables induced by the collisions.

Assuming that the velocity differences may be calculated *a priori*, and choosing a distribution density in an analytical form, one obtains formulas for the evolution of the mean radius and of the relative mean square deviation as a function of the abscissa value along the nozzle axis. The nozzle geometry has no effect, and the agglomeration develops mainly around the throat.

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Main Notation

a, b	density distribution parameters;
A, A_c	normal cross section area of the nozzle, at the throat;
f', q'_p	force and fictitious thermal power induced by collisions;
C^*	characteristic velocity;
K	mass fraction of the condensed phase;
m_p	mass of one particle;
$\overset{\circ}{m}$	mass flowrate of the gaseous phase;
n_p	local particle concentration;
p_0	chamber pressure;
f	particle distribution density;
r, r_m, r_{ms}, r_{mv}	radius, average radius, — for the surface — for the volume;
T_p, T'_p	average temperature of particles, true temperature;
u_p, u'_p, $u^{(0)}$	average velocity of particles, true velocity, flow velocity for equilibrium;
x	abscissa taken along the nozzle axis;
$\bar{\gamma}$	isentropic exponent of the flow equivalent to equilibrium;
ρ_e	volume mass of liquid alumina;
μ	gas viscosity.

The subscript 0 corresponds to initial conditions, the subscript (0) corresponds to equilibrium flow,and the **overlined** quantities correspond to flow equivalent to equilibrium.

I. Introduction

By adding a metal to a solid propellant, the combustion temperature is increased as is the performance. Aluminum is a metal which is most widely used and the optimum performance is achieved for a mass fraction of the metal in the propellant which is relatively large, even though the flow in a nozzle with metallized propellant is the flow of a biphase mixture in which the mass fraction of the condensed oxide is on the order of 0.4.

Capture tests carried out in a jet have shown that the condensed phase is a dispersion of very fine particles whose radius is on the order of 1 μ. The theoretical performances predicted by thermodynamic calculations are never achieved in practice and one can attribute this loss partly to the irreversible exchanges between the gaseous phases and the condensed phases. Therefore, in order to predict the performance of a metallized propellant and to best design a nozzle, it is necessary to accurately describe the behavior of condensed particles in the flow.

One finds that the particle population is distributed according to particle radius. Since it is easy to see that the acceleration of a particle by the gaseous flow is proportional to the reciprocal of the square of its radius, the particles receive different velocities and are subjected to numerous collisions.

Alumina has a temperature close to 3200° K in the combustion chamber and is in the liquid state. It solidifies around 2300° K, i.e., in the divergent part of the nozzle. The particles are, therefore, liquid for quite some time and then they agglomerate due to their collisions. Their average mass tends to increase. This phenomenon is, therefore, susceptible to reducing the performance and it is important to determine its importance.

Not much work has been done on this subject. Marble [1] established equations which describe the variation of the particle distribution. A formula is derived which gives the growth of the average radius along the nozzle abscissa and the influence of the operational parameters. Crowe and Willoughby [2] numerically determined the increase of the average radius in the volume and studied the influence of various parameters.

The equations in this report will be established using a different formal base than that used by Marble. The formulas will be applied later on and we will establish more accurate conclusions on that occasion.

II. Mathematical Model — Hypotheses

We will assume that the particles are spheres characterized by their radius. This hypothesis is not very realistic because an accelerated droplet is deformed because of the action of stresses which are applied to its surface and the resulting internal circulation. Since it is difficult to predict the shape of a droplet at any time during its history, and therefore, the associated drag, and since the study of collisions of such particles would be particularly difficult, we are forced to assume this hypothesis. From the same point of view, we will neglect the problem of particle stability [3] and we will assume that all the collisions are binary ones and only result in the production of a single particle.

We will assume that the gaseous flow is not perturbed locally by collisions, which allows one to calculate the drag of a particle independent from all of the others. This implies that the average stay time of a particle in the vicinity of another particle is very much smaller than the average time between collisions.

We will assume that the flow is steady, one-dimensional in any cross section. All the velocities are, therefore, parallel to the nozzle axis of revolution. This hypothesis is coherent with the collision mechanism: two particles having velocities parallel to the axis will produce a new particle having a velocity parallel to the axis, due to the conservation of momentum.

We will define two functions of the abscissa value x and the radius r which are continuous: $u_p(r, x)$ and $T_p(r, x)$, which are the average velocities and temperatures of the particles having the same radius. These quantities are different from the true velocities and temperatures, which are subjected to numerous discontinuities, and in all strictness, depend on the complete history of a particle.

These two latter hypotheses imply that the Boltzmann equation cannot be used here. The integration of this equation would be difficult.

The population of the particles will be locally described by the distribution density f, (r, x), which is a continuous function of r and x. We assume that it exists based on the large number of particles.

Because of these hypotheses, we are forced to describe the condensed phase as a continuous medium. Our objective will now be to find the variation of u_p, T_p, and principally f.

III. Variation of the Distribution Density as a Function of Abscissa Value

Let us consider the n_p particles at an instant t contained in a unit volume. We will study the number of collisions between the particles of radius a and those of radius b during an infinitesimal increase in time dt.

— The number of collisions of one particle of radius a with all the other particles of radius b is given by the following elementary formula:

[illegible]

where $\pi(a + b)^2$ is the collision cross section, $n_p f(b, x)db$ is the number of particles b, $u_p(a) - u_p(b)$ is the relative velocity between particles a and b.

— The number of collisions between the two particle classes can be written as:

[illegible]

and all the collisions produced particles whose radius is obtained by writing down the equation for conservation of particle mass: $(a^3 + b^3)^{1/3}$.

The number of particles of radius r which disappear because of the collision is, therefore,

[illegible]

The integral in the second term will be called $I_1(r, x)$.

All the collisions produced particles of a radius greater than r.

Let us now find the number of collisions which the particles having radii between r and r + dr produce from smaller particles a and b. The integration range D is found if we take the relationship: $a^3 + b^3 = r^3$ into account.

From this, we find

[illegible]

The coefficient 1/2 is introduced in order to take into account the symmetry property of a and b in the integral.

If we integrate with respect to B and hold a constant, we find:

[illegible]

The integral in the second term will be called $I_2(r, x)$.

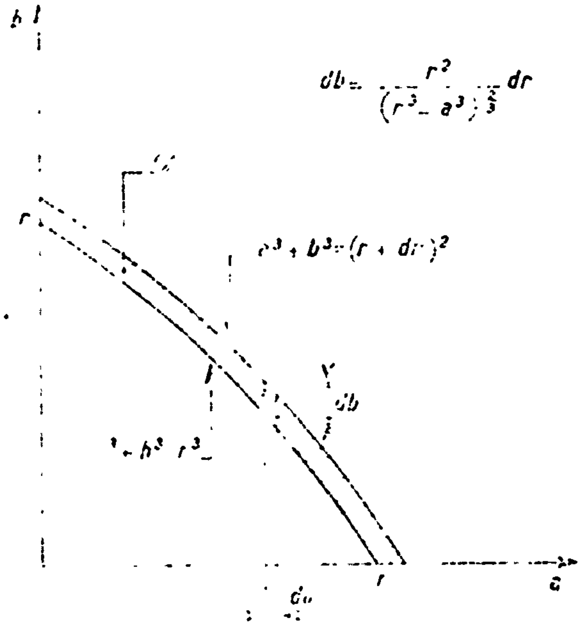

Figure 1. Integration region for calculating particles formed by agglomeration.

Remark

It is easy to verify that the total number of collisions is twice the number of particles formed. Each collision produces the disappearance of one particle. Therefore, we have

[illegible]

by changing the variable in the second integral.

Let us now establish the balance sheet of the particles of the class (r, r + dr).

At the time t + dt, we have:

[illegible]

particles of radius r and in this expression, we can replace dt by $(dx)/[u_p(r)]$.

If we take into account the normalization condition for the distribution density, we fi.d:

[illegible]

Since

$$f(r, x + dx) \simeq f(r, x) + \frac{\partial f}{\partial x} dx,$$

we can, from the preceding equation, derive $(\partial f/\partial x)dx$ and then $\partial f/\partial x$ by letting dx go to zero. Finally, we find:

$$\boxed{\frac{\partial f}{\partial x} = \pi u_p \left[\frac{1}{u_r} (\cdots f I_1 - r^2 I_2) - f \int_0^\infty (\cdots f I_1 - r^2 I_2) \frac{dr}{u_r} \right]}$$

We can easily verify the following relationship:

$$\int_0^\infty \frac{\partial f}{\partial x} dr = \frac{\partial}{\partial x} \int_0^\infty f(r, x)\, dr = 0.$$

This equation describes the evolution of the distribution density of particles. It is related to the particle velocity in a complex way and we cannot consider a numerical calculation without making it discrete.

IV. Equations for One Particle

The equation of motion and the thermal equation of a particle do not only contain the classical terms corresponding to the drag and the thermal exchange with the gas, but they also contain an additional collision term.

In effect, the velocity or the temperature of a particle of radius r, which is the result of the agglomeration of two particles which are smaller, is different from thevelocity or temperature of the same particles which have not experienced collisions. It is assumed that this difference is instantaneously distributed over other particles.

This additional term must be zero on the average, because we are dealing with an interaction.

In order to evaluate this additional term, it is necessary to know the true velocity or temperature of each particle before the collision and, from this, to derive the velocity or temperature of the particle formed after collision. This true quantity depends on the entire history of the particle. It is not possible to exactly know it except in the case where every particle reaches its asymptotic regime before the collision (this nomenclature is developed in [3]: hypothesis of "weak slip," frequently used by Marble is related to this). If this is not the case, it is possible to establish the existence of a correction or defect factor to be used for the true velocity or the average velocity.

The momentum of a particle produced by the agglomeration of a particle a and of a particle b is:

[illegible]

where u'_p designates the true velocity before collision.

The momentum of all the particles of radius r formed in this way per unit of volume and during a time dt is, therefore, given by:

[illegible]

The integral in the second term will be called $I_3(r, x)$.

The variation in the momentum of the particles r only produced by collisions is, therefore:

[illegible]

therefore,

$$d\Delta = \frac{4}{3}\pi\rho_e \pi n_p^2 \, dr\, dt \left(r^2 I_2 - r^3 u_p'(r) f(r, x) I_1\right).$$

Since the resulting force applied to the particles r is given by:

$$\lim_{dt \to 0} \frac{d\Delta}{dt},$$

it is possible to calculate the elementary force which acts on a particle. It is given by the following expression:

$$\boxed{f_p' = \frac{\frac{4}{3}\pi^2 \rho_e n_p}{f(r, x)} \left(r^2 I_2 - f(r, x)\, r^3 u_p'(r)\, I_1\right)}.$$

It is easy to verify that f'_p is zero on the average:

$$\int_0^\infty f_p' f(r, x)\, dr = 0$$

that is,

$$\int_0^\infty r^2 I_2 \, dr = \int_0^\infty f(r, x)\, r^3 u_p'(r)\, I_1 \, dr$$

which can be obtained by changing variables and from symmetry considerations in the first integral.

The same calculation can be carried out in order to find the fictitious elementary power q'_p related to collisions and relative to one particle:

$$\boxed{q_p' = \frac{\frac{4}{3}\pi^2 \rho_e n_p}{f(r, x)} \left(r^2 I_2 - f(r, x)\, r^3 T_p'(r)\, I_1\right)}$$

with

$$I_2 = \int_0^r \frac{1}{2}\left[a + (r^3 - a^3)^{\frac{1}{3}}\right]^2 f(a, x) f\left((r^3 - a^3)^{\frac{1}{3}}, x\right) \frac{\left|u_p(a) - u_p\left((r^3 - a^3)^{\frac{1}{3}}\right)\right|}{(r^3 - a^3)^{\frac{2}{3}}} \cdot \left[a^3 T_p'(a) + (r^3 - a^3) T_p'\left((r^3 - a^3)^{\frac{1}{3}}\right)\right] da.$$

One can verify that q'_p is zero on the average.

We will now use a few simplifications to find the variation of the distribution density as a function of abscissa.

These simplifications are applied to the velocities. The principle consists of assuming that the particles are sufficiently small and that the flow has a sufficiently small acceleration so that we have:

$$\frac{2r^2\rho_c}{9\mu}\frac{du}{dr} \ll 1 \quad \text{(hypothesis of weak slip).}$$

One then shows [3] that the velocity of a particle is expressed by:

$$u_p = u^{(0)}\left(1 - \frac{2r^2\rho_c}{9\mu}\frac{du^{(0)}}{dr}\right),$$

$u^{(0)}$ is the fictitious flow velocity for which kinetic and thermal equilibrium of the two phases would be realized. This flow corresponds to a perfect gas flow and $u^{(0)}$ does not depend on the cross section considered.

We will now write the velocity differences:

$$|u_p(a) - u_p(b)| = \frac{2}{9}\frac{\rho_e}{\mu} u^{(0)} \frac{du^{(0)}}{dx}(b^2 - a^2)$$

and we will replace $u_p(r)$ by $u^{(0)}$ so that only one particle velocity appears.

Under these conditions, the integrals I_1 and I_2 become:

$$I_1 = \frac{2\rho_c}{9\mu} u^{(0)} \frac{du^{(0)}}{dx} \int_0^\infty (r + a)^2 f(a, x)\, |a^2 - r^2|\, da$$

$$I_2 = \frac{2\rho_c}{9\mu} u^{(0)} \frac{du^{(0)}}{dr} \int_0^r \left(a + (r^3 - a^3)^{1/3}\right)^2 f(a, x)\, f\left((r^3 - a^3)^{1/3}, x\right) \frac{|(r^3 - a^3)^{2/3} - a^2|}{(r^3 - a^3)^{2/3}}\, da.$$

As before, we can express the mass flowrate of the condensed phase by:

$$K\dot{m}^{o} = A \int_0^\infty n_p f(r,x)\, m_p(r)\, u_p(r)\, dr,$$

the expression for n_p becomes

$$n_p \simeq \frac{K\dot{m}^{o}}{\frac{4}{3}\pi \rho_e A u^{(o)}} \times \frac{1}{\int_0^\infty f(r,x)\, r^3 dr}.$$

Substituting this in ∂f/∂x, we obtain finally:

$$\frac{\partial f}{\partial x} \simeq \frac{K\dot{m}^{o}}{\frac{4}{3}\pi \rho_e A u^{(o)2}} \times$$

$$-f(r,x)\, I_1 + r^2 I_2 - f(r,x) \frac{\int_0^\infty \left(-f(r,x)\, I_1 + r^2 I_2\right) dr}{\int_0^\infty f(r,x)\, r^3 dr}.$$

V. Variation of the Characteristic Parameters of the Distribution

One distribution law can be described by a reduced number of parameters: average value, standard deviation, shape coefficients.

By definition, the average radius and average standard deviation are given by:

$$r_m(x) = \int_0^\infty r f(r,x)\, dr$$

$$\sigma^2(x) = \int_0^\infty (r - r_m)^2 f(r,x)\, dr.$$

By taking the derivative under the sum sign, we immediately find:

$$\frac{dr_m}{dx} = \int_0^\infty r \frac{\partial f}{\partial x}\, dr$$

$$\frac{d\sigma^2}{dx} = \int_0^\infty (r - r_m)^2 \frac{\partial f}{\partial x}\, dr.$$

This last relationship can be written as:

$$\frac{d\sigma^2}{dx} + 2 r_m \frac{dr_m}{dx} = \int_0^\infty r^2 \frac{\partial f}{\partial x}\, dr.$$

In particular, let us calculate the integrals in the second terms:

$$-\int_0^x r \frac{\partial f}{\partial r} dr = \frac{K\dot{m}}{\frac{4}{3}\rho_p \Lambda u^{(0)2}} \frac{\int_0^x \left[-fI_1 + r^2 I_2 - f(r,x)\int_0^x (-fI_1 + r^2 I_2)\, dr\right] r dr}{\int_0^x f r^3 dr}$$

The integral in the numerator is transformed into:

$$\int_0^x (-fI_1 + r^2 I_2)(r - r_m)\, dr.$$

Since we know that:

$$\int_0^r fI_1 dr = 2\int_0^r r^2 I_2 dr$$

we find:

$$\int_0^r (-fI_1 + r^2 I_2)(r - r_m)\, dr = -\int_0^r r f I_1 dr + \int_0^r r^3 I_2 dr + \frac{r_m}{2}\int_0^r f I_1 dr.$$

By replacing I_1 and I_2 by their respective expressions, we obtain

$$\int_0^r r f I_1 dr = \iint_D (y+z)^2 f(y,x) f(z,x) [u_p(y) - u_p(z)]\, y\, dy\, dz.$$

D_0 is the first quadrant in the (y, z) plane and, therefore, this integral can also be written as:

$$\iint_D (y+z)^2 f(y,x) f(z,x) [u_p(y) - u_p(z)]\, z\, dy\, dz$$

or also:

$$\frac{1}{2}\iint_D (y+z)^2 f(y,x) f(z,x) [u_p(y) - u_p(z)] (y+z)\, dy\, dz,$$

$$\int_0^r f I_1 dr = \iint_D (y+z)^2 f(y,x) f(z,x) [u_p(y) - u_p(z)]\, dy\, dz)$$

$$\int_0^r r^3 I_2 dr = \iint_{D'} \frac{1}{2} r^3 \left(a^3 + (r^3 - a^3)^{\frac{1}{3}}\right)^2 f(a,x)\, f\left((r^3 - a^3)^{\frac{1}{3}}, x\right) \frac{[u_p(a) - u_p(r^3 - a^3)^{\frac{1}{3}}]}{(r^3 - a^3)^{\frac{1}{3}}}\, da\, dr.$$

D' is between the r axis and the first bisector in the plane (a, r).

A change in variables:

$$y = (r^2 - a^2)^{1/2}$$
$$z = a,$$

transforms this integral into

$$\iint_{D'} \frac{1}{2}(y+z)^2 f(y, x) f(z, x) |u_p(y) - u_p(z)| (y^2 + z^2)^{1/2}\, dy\,dz.$$

Therefore, we finally have

$$\int_0^{\infty} r f (I_1 - r^2 I_2)(r - r_m)\, dr = \frac{1}{2}\iint_{D'} (y+z)^2 f(y, x) f(z, x) |u_p(y) - u_p(z)| \left((y^2 + z^2)^{1/2} - (y + z - r_m)\right) dy\,dz.$$

Because of symmetry with respect to the first bisector, this integral is transformed into:

$$\iint_{D''} (y+z)^2 f(y, x) f(z, x) \left(u_p(z) - u_p(y)\right) \left((y^2 + z^2)^{1/2} - (y + z - r_m)\right) dy\,dz.$$

It is advantageous to introduce the following reduced variables:

$$Y = \frac{y}{r_m}, \quad Z = \frac{z}{r_m}$$

which finally leads to these expressions for the integral

$$\int_0^{\infty} r \frac{\partial f}{\partial t}\, dr = \frac{K r_m^{6}}{\frac{4}{3}\pi \rho N(t)} \cdot \frac{\iint_{\Delta} (Y+Z)^2 f(Y, x) f(Z, x) \left(u_p(Z, x) - u_p(Y, x)\right) \times \left((Y^2 + Z^2)^{1/2} - (Y + Z - 1)\right) dY\,dZ}{\int_0^{x} f(Y, x)\, Y^3\, dY}$$

The calculation is carried out in the same way for

$$\int_0^\infty r^3 \frac{\partial f}{\partial x}\, dr = \frac{K\mathring{m}}{\frac{4}{3}\rho_p \Lambda u^{(0)2}}\, r_m^3 \frac{\displaystyle\iint_0^\infty (Y+Z)^2 f(Y,x) f(Z,x) \left(u_p(Z,x) - u_p(Y,x)\right) \left((Y^3+Z^3)^{\frac{1}{3}} - (Y+Z)^2 + 1 + \frac{\sigma^2}{r_m^2}\right) dY\, dZ}{\displaystyle\int_0^\infty f(Y,x)\, Y^3\, dY}$$

Let us now use the hypothesis of small slip for calculating the velocity differences. We obtain the following differential equations

$$\frac{dr_m}{dx} = \frac{K\mathring{m}}{6\Lambda\sqrt{2}} \frac{1}{u^{(0)}} \frac{du^{(0)}}{dx}\, r_m^3 \frac{\displaystyle\iint_0^\infty (Y+Z)^2 (Y^2 - Z^2) f(Y,x) f(Z,x) \left((Y^3+Z^3)^{\frac{1}{3}} - (Y+Z) + 1\right) dY\, dZ}{\displaystyle\int_0^\infty f(Y,x)\, Y^3\, dY}$$

$$\frac{d\sigma^2}{dx} + 2 r_m \frac{dr_m}{dx} = \frac{K\mathring{m}}{6\Lambda\sqrt{2}} \frac{1}{u^{(0)}} \frac{du^{(0)}}{dx}\, r_m^4 \frac{\displaystyle\iint_0^\infty (Y+Z)^2 (Y^2 - Z^2) f(Y,x) f(Z,x) \left((Y^3+Z^3)^{\frac{2}{3}} - (Y^3+Z^3) - 1 + \frac{\sigma^2}{r_m^2}\right) dY\, dZ}{\displaystyle\int_0^\infty f(Y,x)\, Y^3\, dY}$$

The calculation of the integrals, therefore, means it is necessary to choose an analytic form for the distribution density.

VI. First Approximation. Utilization of a Similitude Law

Let us study laws of the form

$$f(r,x) = k(x) \cdot g\left(\frac{r}{r_m}\right).$$

The normalization condition leads to:

$$k(x) = \frac{1}{r_m}$$

and it is convenient to select g(X) such that $\int_0^\infty g(X)\, dX = 1$.

The definition of the average radius results in the condition

$$\int_0^\infty X g(X)\, dX = 1.$$

It is easy to see that σ is proportional to r_m:

$$\sigma = r_m \left(\int_0^\infty X^2 g(X)\, dX - 1 \right)^{\frac{1}{2}}$$

and $g(X)$ must satisfy $\int_0^\infty X^2 g(X)\, dX > 1.$

Fein [4] showed that the particles are distributed according to an exponential law in the combustion chamber. In addition, the results of captured tests of particles in jets carried out at the ONERA have shown the law to be of the form:

$$f(r) = k r^b e^{-ar}.$$

Here, we will assume the law

$$f(r, t) = \frac{a^{b+1}}{\Gamma(b+1)} r^b e^{-ar}$$

defined for $a > 0,\ b > -1.$

From this law, we can derive the following average radius and standard deviation:

$$r_m = \frac{b+1}{a}$$

$$\sigma^2 = \frac{b+1}{a^2}$$

such that

$$f\left(\frac{r}{r_m}, t\right) = \frac{1}{r_m} \frac{(b+1)^{b+1}}{\Gamma(b+1)} \left(\frac{r}{r_m}\right)^b e^{-(b+1)\frac{r}{r_m}}.$$

We will assume that b is constant in this first approximation and we will calculate the evolution of the average radius.

From this law, we are able to calculate the integral:

$$\int_0^\infty f(X, t)\, X^3 dX = \frac{1}{r_m} \frac{(b+1)^{b+1}}{\Gamma(b+1)} \int_0^\infty Y^{b+3} e^{-(b+1)Y}\, dY$$
$$= \frac{1}{r_m} \frac{\Gamma(b+4)}{(b+1)^3\, \Gamma(b+1)}.$$

such that the differential equation with respect to r_m becomes:

$$\frac{dr_m}{dt} = \frac{K\dot{m}}{6\Lambda_c\mu}\frac{1}{u^{(0)}}\frac{du^{(0)}}{dx} r_m^2 \frac{(b+1)^{2b+5}}{\Gamma(b+1)\,\Gamma(b+4)}$$
$$\iint_{D^*} (Y+Z)^2(Y^2-Z^2)Y^bZ^b e^{-(b+1)(Y+Z)}$$
$$\times\left((Y^3-Z^3)^{\frac{1}{3}}-(Y+Z)+1\right)dY\,dZ.$$

We will set

$$J_1(b) = \frac{(b+1)^{2b+5}}{\Gamma(b+1)\,\Gamma(b+4)}$$
$$\iint_{D^*} (Y+Z)^2(Y^2-Z^2)Y^bZ^b e^{-(b+1)(X+Y)}$$
$$\left((Y^3-Z^3)^{\frac{1}{3}}-(Y-Z)+1\right)dY\,dZ$$

in the preceding differential equation. $J_1(b)$ is a constant.

Therefore, the integration becomes possible. Let us set:

$$z = \frac{1}{6}\frac{K\dot{m}\,r_{m0}}{\Lambda_c\mu}\int_0^x \frac{\Lambda_2}{\Lambda}\frac{1}{u^{(0)}}\frac{du^{(0)}}{dx}\,dx.$$

we find

$$\frac{d}{dz}\left(\frac{r_m}{r_{m0}}\right) = J_1\left(\frac{r_m}{r_{m0}}\right)^2$$

$$\boxed{r_m = \frac{r_{m0}}{1-J_1 z}}.$$

Except for the coefficient, the formula is identical to that found by Marble.

Interpretation

The function z is the product of a constant and an abscissa function.

This constant can also be written as:

$$\frac{1}{6}\frac{K\dot{m}\,r_{m0}}{\Lambda_c\mu} = \frac{1}{6}\frac{Kp_0}{C^*\mu}$$

and characterizes the propellant and the operational conditions. The influence of the mass fraction of the condensed phase, the characteristic velocity for the gas viscosity (which accelerates the particles) and the operational pressure are clearly demonstrated.

The abscissa function in reality is a universal function of the cross section ($\bar{\gamma}$ fixed) and of the same $u^{(0)}$ by definition: the nozzle geometry does not occur explicitly. In effect, we have [3]:

$$\int_{u_0^{(0)}}^{u^{(0)}} \frac{A_c}{A} \frac{du^{(0)}}{u^{(0)}} = \frac{\bar{\gamma}}{\bar{\gamma} - 1} \int_{\bar{m}^{(0)}}^{\bar{m}_0^{(0)}} \frac{(\bar{\gamma} z)^{\frac{1}{\bar{\gamma}-1}}}{\left(\dfrac{1 + \frac{2}{\bar{\gamma}-1} z}{\frac{\bar{\gamma}+1}{\bar{\gamma}-1}}\right)^{1 + \frac{\bar{\gamma}+1}{2(\bar{\gamma}-1)}}} dz$$

with

$$\frac{A}{A_c} = \frac{\left(\dfrac{1 + \frac{2}{\bar{\gamma}-1} \bar{m}^{(0)}}{\frac{\bar{\gamma}+1}{\bar{\gamma}-1}}\right)^{\frac{\bar{\gamma}+1}{2(\bar{\gamma}-1)}}}{(\bar{\gamma}\bar{m}^{(0)})^{\frac{1}{\bar{\gamma}-1}}}.$$

It is easy to show that the preceding integral is limited when [illegible], i.e., when the equivalent flow Mach number tends to infinity.

The preceding function is shown as a function of x in Figure 2 for a given nozzle profile. It can be seen that the function increases rapidly in the vicinity of the throat and then strives towards a constant value.

Calculation of J_1

By using polar coordinates, it becomes easier to calculate the integral

$$\iint_{D} (Y^2 + Z^2)(Y^2 - Z^2) Y^6 Z^6 e^{[illegible]} \left((Y^3 + Z^3) - (Y + Z) - 1\right) dY\,dZ$$

$$= \int_0^{[illegible]} \int_0^{[illegible]} R^{[illegible]} (\cos\theta + \sin\theta)^2 (\cos^2\theta - \sin^2\theta) \cos^6\theta \sin^6\theta \; e^{[illegible]} \left(R(\cos^3\theta + \sin^3\theta) - R(\cos\theta + \sin\theta) - 1\right) dR\,d\theta .$$

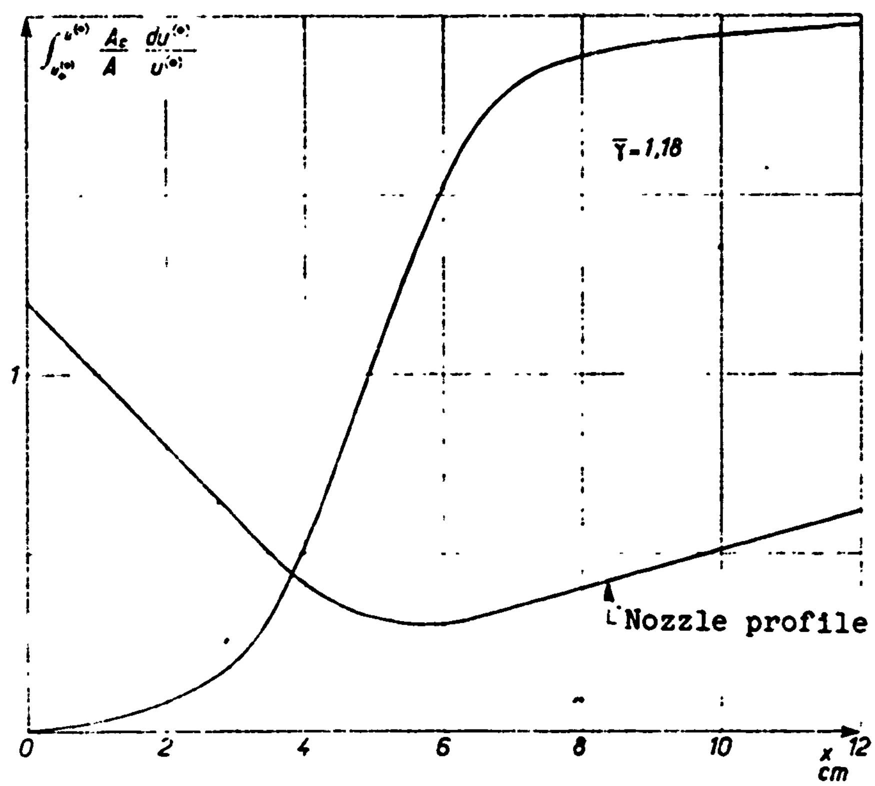

Figure 2. Universal abscissa function.

The integration with respect to R can be carried out and, after simplification, we find:

$$J_1 = \frac{\Gamma(2b+6)}{(b+1)\,\Gamma(b+2)\,\Gamma(b+4)}$$
$$\int_0^{\frac{\pi}{4}} \frac{(\cos\theta+\sin\theta)\cos^b\theta\sin^b\theta}{(\cos\theta-\sin\theta)^{2b+3}}$$
$$\times\left[(2b+6)\,\frac{(\cos^3\theta+\sin^3\theta)^{\frac{1}{2}}}{\cos\theta+\sin\theta}-(b+5)\right]d\theta.$$

Tranforming to the sine of the double arc, we find:

$$J_1 = \frac{\Gamma(2b+6)}{(b+1)\,\Gamma(b+2)\,\Gamma(b+4)}\Big((2b+6)\,K_1(b)-(b+5)\,K_2(b)\Big)$$

or

$$K_1(b) = \frac{1}{2^{b+1}}\int_0^1 \frac{x^b}{(1-x)^{b+2}}\,dx$$
$$K_2(b) = \frac{1}{2^{b+\frac{3}{2}}}\int_0^1 \frac{x^b(2-x)^{\frac{1}{2}}}{(1-x)^{b+\frac{5}{2}}}\,dx.$$

A new change in variable finally results in

$$J_1(b) = \frac{\Gamma(2b+6)}{(b+1)\,\Gamma(b+2)\,\Gamma(b+4)\,2^{2b+2}} \left[(2b+6)\,I'_1 - \frac{b+5}{b+1}\right].$$

where

$$I_1 = \int_0^1 t^b \left(1 - \frac{3}{4}t\right)^{\frac{1}{3}} dt.$$

First of all, we should note that I'_1 can be calculated for $b = 0$

$$I'_1(0) = 1 - \frac{1}{4\sqrt[3]{4}}.$$

We obtain the following recursion formula:

$$I'_1(b+1) = \frac{4(b+1)\,I'_1(b) - \frac{1}{\sqrt[3]{4}}}{3b+7} \text{ for } b > -1.$$

It is therefore sufficient to calculate I'_1 in the intervals 0 and 1. The numerical calculation is carried out by expanding the quantity:

$$\left(1 - \frac{3}{4}t\right)^{\frac{1}{3}}$$

according to integral powers of t, which is possible for t between 0 and 1.

Therefore, we find

$$I_1(b) = \frac{1}{b+1} + \sum_{n=1}^{\infty} (-1)^n \frac{1\,.\,(1-3)\,\ldots\,(1-3(n-1))}{2^{2n}\,n\,!} \times \frac{1}{b+n+1}.$$

In this last sum, all the terms are negative. We can verify that this series is convergent using the D'Alembert criterion.

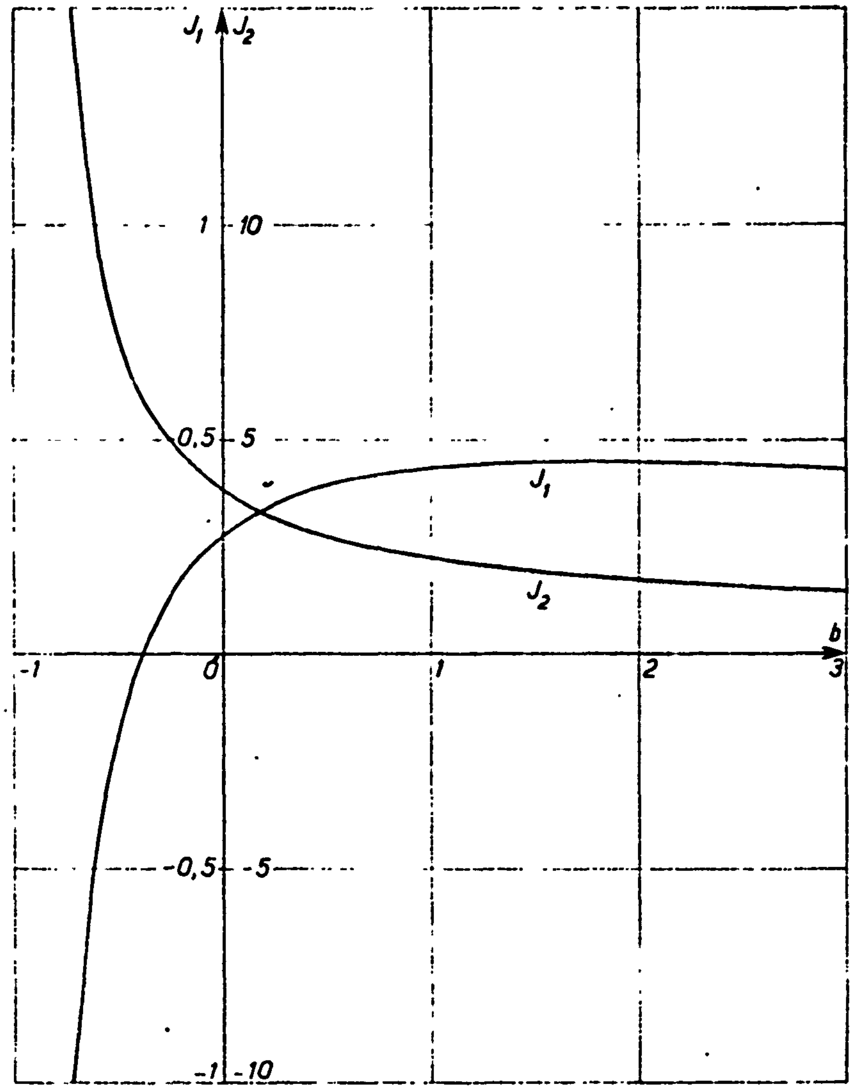

Figure 3. The functions $J_1(b)$ and $J_2(b)$.

The function $J_1(b)$ is given in Figure 3. It can be seen that J_1 is negative for $b < -0.36$, which corresponds to a decrease in the average radius with abscissa value.

We will define an average surface radius and an average volume radius using the formulas

$$r_{m_s} = \left(\int_0^\infty r^2 f\,dr\right)^{1/2}$$
$$r_{m_v} = \left(\int_0^\infty r^3 f\,dr\right)^{1/3}$$

and it is easy to show that:

$$r_{m_s} = r_m \left(\frac{b+2}{b+1}\right)^{\frac{1}{2}}$$

$$r_{m_v} = r_m \left(\frac{(b+3)(b+2)}{(b+1)^2}\right)^{\frac{1}{3}}.$$

Therefore, it is necessary to assume that the average volume of the particles can decrease for certain shapes of the distribution law, which is a contradiction to the agglomeration phenomenon: *therefore, this analysis is insufficient.*

VII. Second Approximation

Let us assume the same distribution law, but this time we will assume that b varies with the abscissa value along the nozzle. This law depends now on two variable parameters a and b, or on r_m and σ^2, which are related to these two parameters.

The two equations which define the evolution of the particle population are:

$$\begin{cases} \dfrac{dr_m}{dx} = \dfrac{K\overset{0}{m}}{6 A q} \cdot \dfrac{1}{u^{(0)}} \dfrac{du^{(0)}}{dx} r_m^2 \dfrac{(b+1)^{2b+6}}{\Gamma(b+1)\Gamma(b+4)} \\ \quad \displaystyle\iint_{D^*} (Y+Z)^2 (Y^2 - Z^2) Y^b Z^b e^{-(b+1)(Y+Z)} \left((Y^3+Z^3)^{\frac{1}{3}} - (Y+Z) + 1\right) dY\,dZ \\ \dfrac{d\sigma^2}{dx} + 2r_m \dfrac{dr_m}{dx} = \dfrac{K\overset{0}{m}}{6 A q} \cdot \dfrac{1}{u^{(0)}} \dfrac{du^{(0)}}{dx} r_m^2 \dfrac{(b+1)^{2b+5}}{\Gamma(b+1)\Gamma(b+4)} \\ \quad \displaystyle\iint_{D^*} (Y+Z)^2 (Y^2 - Z^2) Y^b Z^b e^{-(b+1)(Y+Z)} \left((Y^3+Z^3)^{\frac{2}{3}} - (Y^2+Z^2) + 1 + \frac{\sigma^2}{r_m^2}\right) dY\,dZ. \end{cases}$$

Since

$$\frac{\sigma^2}{r_m^2} = \frac{1}{b+1},$$

the grouping of the coefficient of b and of the second integral can be called $J_2(b)$.

We can introduce the same variable z as before, and write:

$$\frac{dr_m}{dz} = \frac{r_m^2}{r_{m0}} J_1(b),$$

$$\frac{d\sigma^2}{dz} + 2r_m \frac{dr_m}{dz} = \frac{r_m^2}{r_{m0}} J_2(b).$$

By expressing r_m and σ^2 as functions of a and b, we find

$$\frac{da}{dz} = -\frac{a_0}{b_0+1}(b+1)\left((2b+3)J_1 - (b+1)J_2\right)$$
$$\frac{db}{dz} = -\frac{a_0}{b_0+1}\frac{(b+1)^2}{a}\left((2b+4)J_1 - (b+1)J_2\right).$$

By combining, we can express a as a function of b:

$$\frac{da}{db} = \frac{a}{b+1} \times \frac{(2b+3)J_1 - (b+1)J_2}{(2b+4)J_1 - (b+1)J_2}$$

Therefore,

$$a = a_0 \exp\left[\int_{b_0}^{b} \frac{(2b+3)J_1 - (b+1)J_2}{(2b+4)J_1 - (b+1)J_2} \times \frac{db}{b+1}\right]$$

and, consequently, we will set:

$$L(b) = \exp\left[\int_{0}^{b} \frac{(2b+3)J_1 - (b+1)J_2}{(2b+4)J_1 - (b+1)J_2} \times \frac{db}{b+1}\right].$$

from which it follows that:

$$a = a_0 \frac{L(b)}{L(b_0)}.$$

By substituting this expression for a in the differential equation for b, we find

$$\frac{db}{dz} = -\frac{L(b_0)}{b_0+1}\frac{(b+1)^2}{L(b)}\left((2b+4)J_1 - (b+1)J_2\right)$$

We will set

$$M(b) = \int_0^b \frac{L(b)}{(2b+4)J_1 - (b+1)J_2} \times \frac{db}{(b+1)^2},$$

from which we find

$$\boxed{z - z_0 = \frac{b_0+1}{L(b_0)}\left(M(b) - M(b_0)\right)}.$$

This relationship defines b as a function of z and makes it possible to calculate the variations in the average radius and the mean square deviation as a function of abscissa value:

$$\boxed{\begin{array}{l} \dfrac{r_m}{r_{m0}} = \dfrac{b+1}{b_0+1} \times \dfrac{L(b_0)}{L(b)} \\ \dfrac{\sigma^2}{r_m^2} = \dfrac{b_0+1}{b+1} \end{array}}.$$

Calculation of J_2

The calculation of J_2 is done just like the calculation of J_1.

After transforming to polar coordinates, we find first of all

$$J_2 = \frac{\Gamma(2b+6)}{(b+1)\,\Gamma(b+2)\,\Gamma(b+4)} \int_0^{\frac{\pi}{4}} \frac{(\cos\theta - \sin\theta)\cos^b\theta\sin^b\theta}{(\cos\theta+\sin\theta)^{2b+3}} \left[\frac{(2b+6)(2b+7)}{b+1}\,\frac{(\cos^3\theta-\sin^3\theta)^{\frac{2}{3}}-1}{(\cos\theta+\sin\theta)^2} + b + 2\right] d\theta$$

then

$$J_2 = \frac{\Gamma(2b+6)}{(b+1)\,\Gamma(b+2)\,\Gamma(b+4)\,2^{2b+2}} \left[\frac{(2b+6)(2b+7)}{b+1}\left(I'_2 - \frac{1}{2(b+2)} - \frac{1}{b+1}\right) + \frac{b+2}{b+1}\right]$$

and then

$$I'_2 = \int_0^1 t^b \left(1 - \frac{3}{4}t\right)^{\frac{2}{3}} dt$$

I'_2 can be calculated for $b = 0$:

$$I'_2(0) = \frac{4}{5}\left(1 - \frac{1}{4\sqrt[3]{16}}\right).$$

We can establish the following recursion formula:

$$I'_2(b+1) = \frac{4(b+1)\,I'_2(b) - \frac{1}{\sqrt[3]{16}}}{3b+8}$$

and one calculates I'_2 between 0 and 1 using the expansion

$$I'_2(b) = \frac{1}{b+1} + \sum_{n=1}^{\infty} (-1)^n \frac{2(2-3)\dots(2-3(n-1))}{2^{2n}\cdot n!} \times \frac{1}{b+n+1}.$$

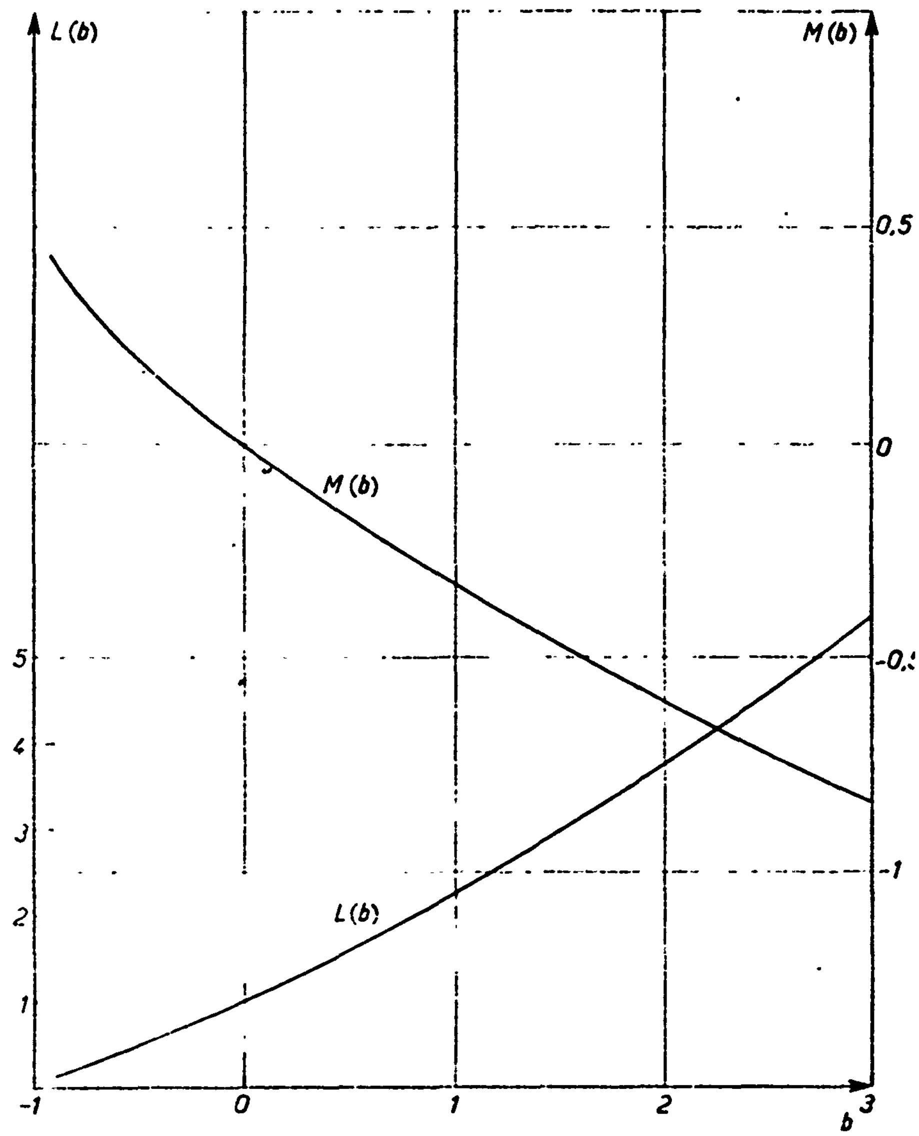

Figure 4. The functions L(b) and M(b).

The function $J_2(b)$ is shown in Figure 3. The functions L(b) and M(b) were calculated numerically and are shown in Figure 4.

Numerical Calculation

The variations in the average radius and the average mean relative standard deviation were calculated for the following conditions:

— biconical nozzle of Figure 1;

— $b_0 = 0$ (exponential law);

— $\frac{K \rho_g r m_0}{C^0 \mu} = 1.$

In Figure 5, we can see that the average radius increases slightly up to the geometric throat and then decreases. The relative size of the distribution increases in a regular way because of the fact that b decreases.

Interpretation

The decrease in the average radius can contradict the ideas usually held regarding the increase in particles; but this phenomenon can be justified.

a) One can verify, first of all, that the average surface radii and average volume radii do increase and that this increase is a maximum in the region of the throat.

b) Let us start with the expression

$$\frac{\partial f}{\partial r} = \frac{K m^0}{\frac{4}{3} \rho_e A u^{(0)2}} \times \frac{(-f I_1 + r^2 I_2) - f \int_0^{x} (-f I_1 - r^2 I_2)\, dr}{\int_0^{x} f r^3 dr}.$$

We will show that the integrals I_1 and I_2 can be calculated when b = 0, i.e., for

$$f(r, x) = \frac{1}{r_m} e^{-R} \quad \text{or} \quad R = \frac{r}{r_m}.$$

In effect, for I_1, we obtain the following:

$$\begin{aligned} I_1 &= \int_0^{r} (r + a)^2 f(a, r) |u_p(r) - u_p(a)|\, da \\ &= \frac{2}{9} \frac{\rho_e}{\mu} u^{(0)} \frac{du^{(0)}}{dx} r_m^4 \int_0^{\infty} (R + t)^2 e^{-t} |R^2 - t^2|\, dt \\ &= \frac{2}{9} \frac{\rho_e}{\mu} u^{(0)} \frac{du^{(0)}}{dx} r_m^4 \left[\int_0^{R} (R + t)^2 (R^2 - t^2) e^{-t} dt \right. \\ &\qquad \left. - \int_R^{\infty} (R + t)^2 (R^2 - t^2) e^{-t} dt \right] \end{aligned}$$

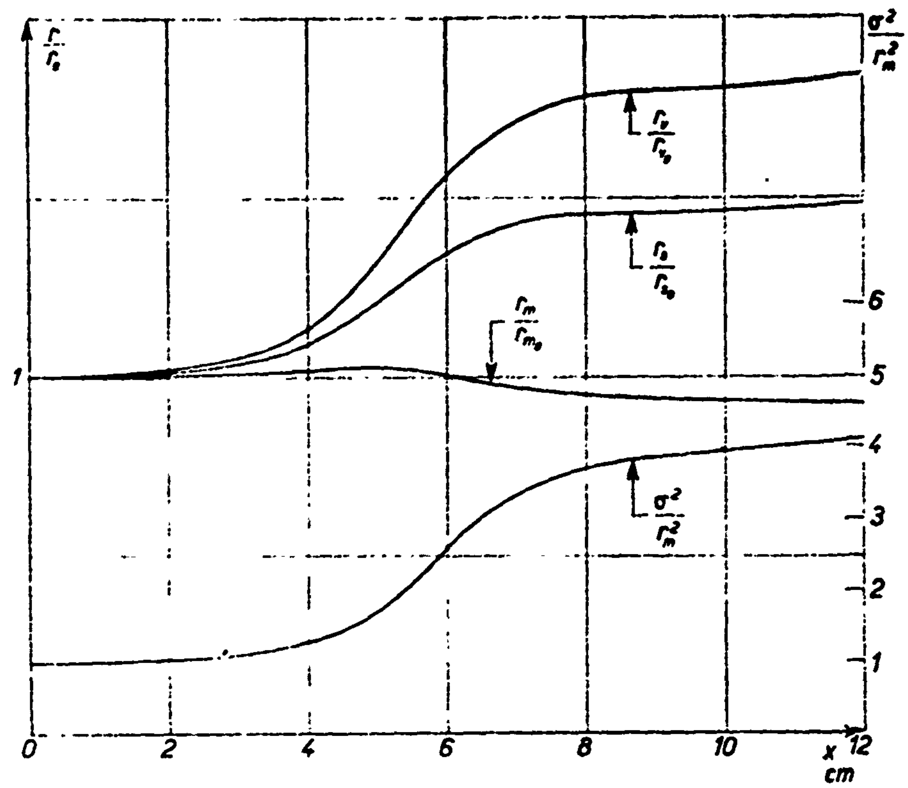

Figure 5. Variation of the particles as a function of the abscissa value.

and after calculating the integrals, we find:

$$I_1 = \frac{2}{9}\frac{\rho_p}{\mu} u^{(0)} \frac{du^{(0)}}{dx} r_m^4$$
$$[R^4 + 2R^3 - 12R - 24 + 8e^{-R}(2R^3 + 6R^2 + 9R + 6)].$$

Also, the calculation of I_2 can be done in the following way:

$$I_2 = \frac{1}{2}\int_0^r [a + (r^3 - a^3)^{\frac{1}{3}}]^2 f(a, x) f\left((r^3 - a^3)^{\frac{1}{3}}, x\right) \frac{|u_p(a) - u_p\left((r^3 - a^3)^{\frac{1}{3}}\right)|}{(r^3 - a^3)^{\frac{2}{3}}} da$$

$$= \frac{2}{9}\frac{\rho_p}{\mu} u^{(0)} \frac{du^{(0)}}{dx} r_m \frac{1}{2} \left[\int_0^{\frac{R}{\sqrt[3]{2}}} \left(t + (R^3 - t^3)^{\frac{1}{3}}\right)^2 \exp\left[-\left(t + (R^3 - t^3)^{\frac{1}{3}}\right)\right] \frac{(R^3 - t^3)^{\frac{2}{3}} - t^2}{(R^3 - t^3)^{\frac{2}{3}}} dt \right.$$
$$\left. - \int_{\frac{R}{\sqrt[3]{2}}}^{r} \left(t + (R^3 - t^3)^{\frac{1}{3}}\right)^2 \exp\left[-\left(t + (R^3 - t^3)^{\frac{1}{3}}\right)\right] \frac{(R^3 - t^3)^{\frac{2}{3}} - t^2}{(R^3 - t^3)^{\frac{2}{3}}} dt\right].$$

It can be seen that, by changing the variable:

$$z^3 = R^3 - t^3,$$

and we find that the two preceding integrals are equal.

Since

$$\frac{d}{dt}\left(t + (R^3 - t^3)^{\frac{1}{3}}\right) = \frac{(R^3 - t^3)^{\frac{2}{3}} - t^2}{(R^3 - t^3)^{\frac{2}{3}}},$$

we find

$$I_2 = \frac{2\,\rho_e}{9\,\mu} u^{(0)} \frac{du^{(0)}}{dx} r_m \int_0^{\frac{2R}{\sqrt[3]{2}}} t^2 e^{-t}\, dt$$

$$= \frac{2\,\rho_e}{9\,\mu} u^{(0)} \frac{du^{(0)}}{dx} r_m$$

$$\left[e^{-R}(R^2 + 2R + 2) - e^{-\sqrt[3]{4}R}\left((\sqrt[3]{4})^2 R^2 + 2\sqrt[3]{4}\,R + 2\right)\right].$$

By substituting I_1 and I_2 in the expression for $\partial f/\partial x$, we finally arrive at the relationship:

$$\frac{\partial f}{\partial x} = \frac{K\dot{m}}{36\sqrt{\mu}} \frac{1}{u} \frac{du}{dx}$$

$$\left[e^{-R}(2R^2 + 12R + 56) - e^{-\sqrt[3]{4}R}\left((\sqrt[3]{4})^2 R^4 - 2\sqrt[3]{4}\,R^3 + 2R^2\right) - 8e^{-2R}(2R^3 + 6R^2 + 9R + 6)\right]$$

$\partial f/\partial x$ is proportional to a certain function F(R) which is shown in Figure 6. We can see that, for $R = 0$, $\partial f/\partial x$ is positive. Therefore, for the selected analytic form, f(0) cannot have any other values than 0($b > 0$), 1($b = 0$), and ∞($b < 0$):f(0) increases starting at 1 and of necessity we find that f(0) is infinite, i.e., for negative b.

$\partial f/\partial x$ is positive for the extreme radius values and is negative for average values. This is to say that the proportion of small and large particles tends to increase, whereas the proportion of average particles tends to decrease: this corresponds very well to an enlargement of the distribution and allows the average radius to increase or decrease.

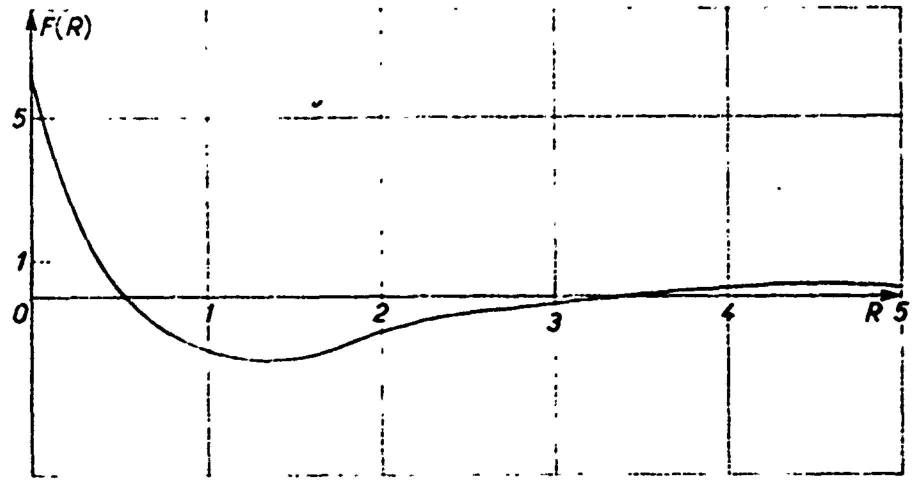

Figure 6. Function ∂f/∂x for b = 0.

This aspect of the phenomenon is due to the collision mechanism which depends not only on the distribution density and the velocity differences, but also on the collision cross section.

c) It remains to interpret the results of capture tests in the jet which result in a density distribution corresponding to b > 0.

First of all, it is necessary to know whether the smallest particles could be detected under a micr.scope and whether or not they have been ignored in the statistics.

One can also wonder whether the samples collected are representative with respect to the real particle population in the jet. In effect, the very small particles follow the streamlines of the gaseous flow around the plate much more than the large ones. The plate is located perpendicular to the nozzle axis and collects the particles. The small particles are subjected to a greater deflection than the large particles and it would be appropriate to correct any experimental law in order to take into account this phenomenon.

The simplified calculations, therefore, result in an adequate representation of the agglomeration phenomenon. This calculation attempts to demonstrate the small role played by the nozzle geometry and to show the simple way in which the operational parameters influence the process. The conclusions which can be derived from the presented formulas qualitatively agree with the results of Crowe and Willoughby [2]. Quantitatively, since the small slip hypothesis was assumed for all particles, the velocity differences and collision frequencies are overestimated by the analysis.

VIII. Conclusion

The agglomeration of alumina particles is therefore primarily translated by an expansion of the distribution law of the particle radius and by an increase in the average particle surface and volume. The collisions bring about a force and a thermal flux for each particle which tends to reduce the velocity differences and temperature differences for particles of different diameters. The agglomeration is primarily important in the throat region. Since the throat has a great influence on the performance of the nozzle, the agglomeration can be an important factor for performance loss. Efforts must be made to utilize the proposed general formulas in numerical calculations,and the particle populations along the nozzle must be studied in experiments.

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